

例 求矩型截面螺绕环电流的磁场分布及螺绕环内的磁通量

解 • 在螺绕环内部做一个环路，可得

$$\oint_L B \cos \theta dl = B \oint_L dl = B \cdot 2\pi r = \mu_0 NI$$

$$B = \mu_0 NI / 2\pi r$$

若螺绕环的截面很小， $r = \bar{r}$

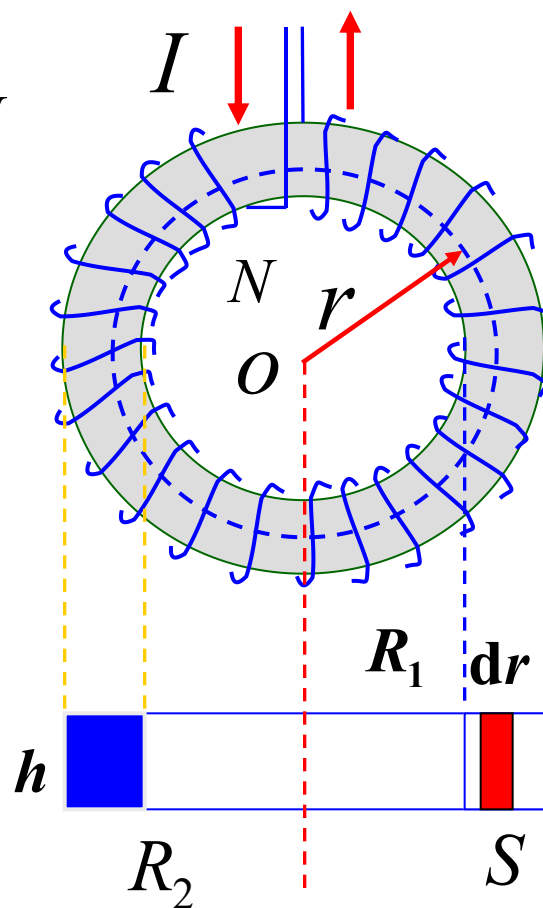
$$B_{\text{内}} = \mu_0 \frac{N}{2\pi \bar{r}} I = \mu_0 n I$$

• 若在外部再做一个环路，可得

$$\sum I_i = 0 \longrightarrow B_{\text{外}} = 0$$

螺绕环内的磁通量为

$$\Phi_m = \int_{R_1}^{R_2} \vec{B} \cdot d\vec{S} = \int_{R_1}^{R_2} \frac{\mu_0 NI}{2\pi r} h dr = \frac{\mu_0 h NI}{2\pi} \ln \frac{R_2}{R_1}$$



§ 10.5 磁场对电流的作用

载流导体产生磁场

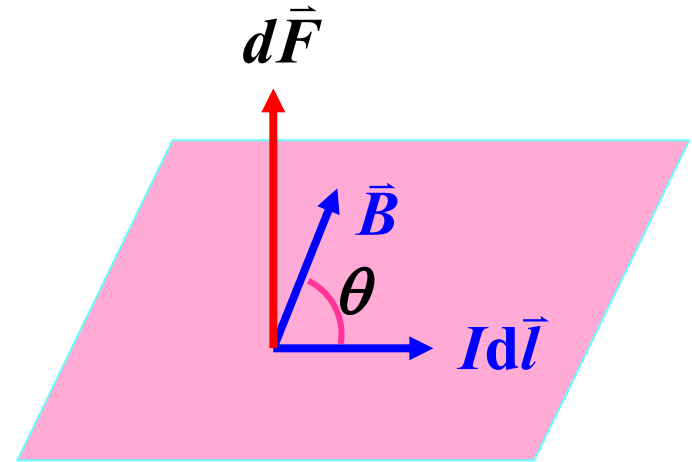


磁场对电流有作用

一. 安培定理

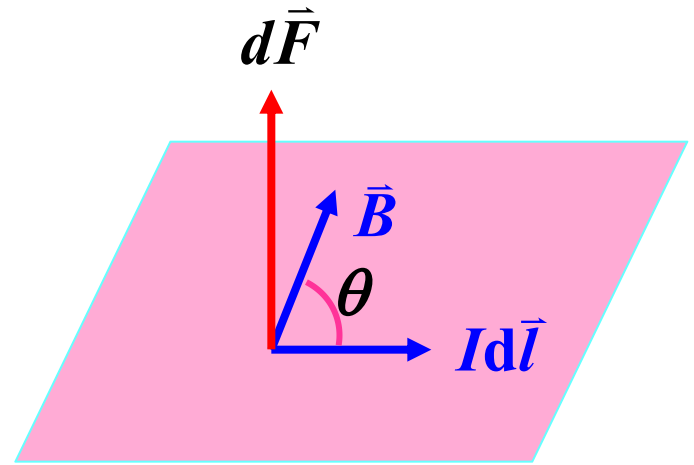
安培力 $d\vec{F} = I d\vec{l} \times \vec{B}$

- 大小: $dF = IdlB \sin \theta$
- 方向: 由右手螺旋法则确定



任意形状载流导线在外磁场中受到的安培力

$$\vec{F} = \int d\vec{F} = \int I d\vec{l} \times \vec{B}$$



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讨论

(1) 安培定理是矢量表述式 $d\vec{F} \Rightarrow dF_x, dF_y, dF_z$

(2) 若磁场为匀强场 $\longrightarrow \vec{F} = \left(\int I d\vec{l} \right) \times \vec{B}$

在匀强磁场中的闭合电流受力 $\longrightarrow \vec{F} = \left(\oint I d\vec{l} \right) \times \vec{B} = 0$

例 在均匀磁场中放置一任意形状的导线，电流强度为 I

求 此段载流导线受的磁力。

解 在电流上任取电流元 $I d\vec{l}$

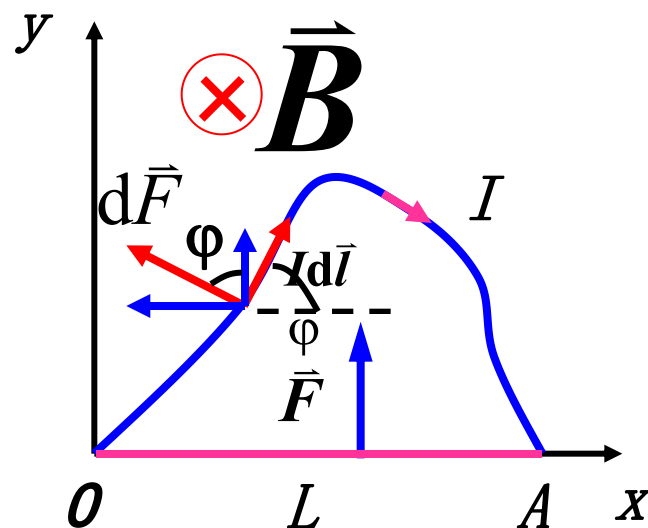
$$d\vec{F} = I d\vec{l} \times \vec{B} = IB d\vec{l}$$

$$dF_x = IB dl \sin \varphi = IB dy$$

$$dF_y = IB dl \cos \varphi = IB dx$$

$$F_x = \int_0^0 IB dy = 0$$

$$F_y = \int_0^L IB dx = IBL$$



相当于载流直导线 $\overline{OA}$ 在匀强磁场中受的力，方向沿 y 向。

例 求一载流导线框在无限长直导线磁场中的受力和运动趋势

解

$$\textcircled{1} \quad f_1 = I_2 b B_1 = I_2 b \frac{\mu_0 I_1}{2\pi a}$$

方向向左

$$\textcircled{3} \quad f_3 = I_2 b B_3 = I_2 b \frac{\mu_0 I_1}{4\pi a}$$

方向向右

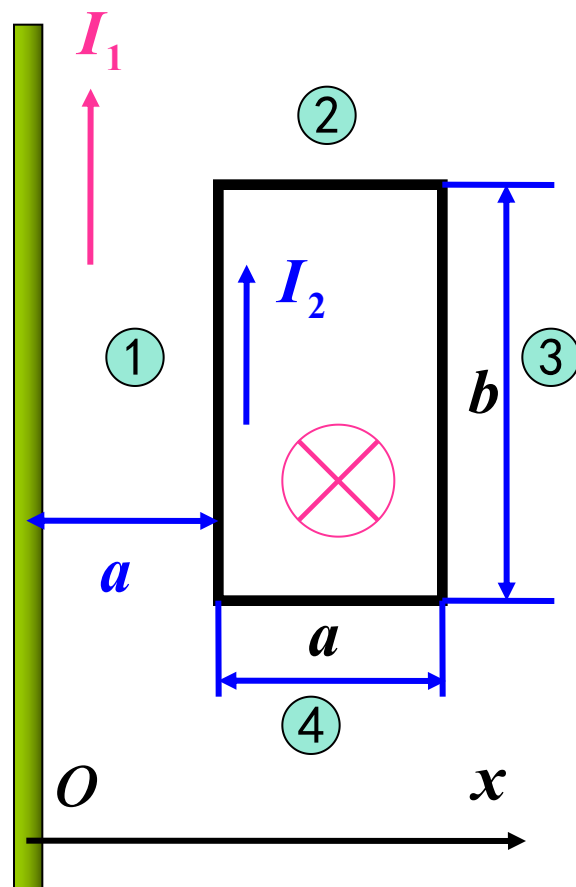
$$\begin{aligned} \textcircled{2} \quad f_2 &= \int_a^{2a} I_2 dl B_1 \sin \frac{\pi}{2} \\ &= \int_a^{2a} \frac{\mu_0 I_1}{2\pi x} I_2 dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln 2 \end{aligned}$$

$$\textcircled{4} \quad f_4 = f_2$$

整个线圈所受的合力: $\vec{F} = \vec{f}_1 + \vec{f}_2 + \vec{f}_3 + \vec{f}_4 = \vec{f}_1 + \vec{f}_3$

$$\because |\vec{f}_1| > |\vec{f}_3|$$

$\therefore$ 线圈向左做平动



二. 磁场对平面载流线圈的作用

1. 在均匀磁场中的刚性矩形载流线圈

(方向相反在同一直线上)

$$F_{DA} = F_{BC}$$

$$F_{CD} = F_{AB} = BIl_2$$

(方向相反不在一条直线上)

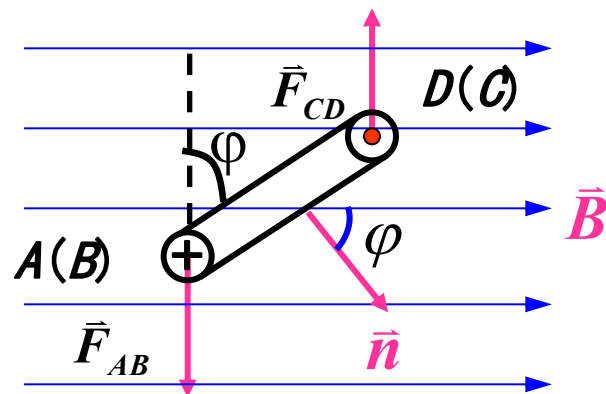
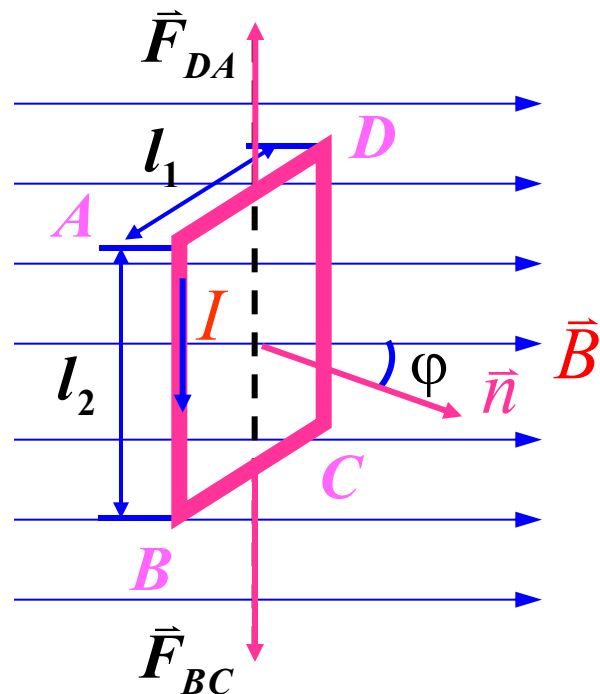
$$\sum \vec{F}_i = 0 \quad (\text{线圈无平动})$$

对中心的力矩为

$$\begin{aligned} M &= F_{AB} \frac{l_1}{2} \sin \varphi + F_{CD} \frac{l_1}{2} \sin \varphi \\ &= l_1 l_2 B I \sin \varphi \end{aligned}$$

$$\text{令 } \vec{S} = S\vec{n} = l_1 l_2 \vec{n} \quad \vec{p}_m = IS\vec{n}$$

线圈若有 N 匝线圈



$$\vec{M} = \vec{p}_m \times \vec{B}$$

$$\vec{M} = N\vec{p}_m \times \vec{B}$$

2. 磁场力的功

$$dA = -M d\varphi = -BIS \sin \varphi d\varphi \quad \text{负号表示力矩作正功时 } \varphi \text{ 减小}$$

$$= Id(BS \cos \varphi) = Id \Phi_m$$

$$A = \int_{\Phi_{m1}}^{\Phi_{m2}} Id \Phi_m = I(\Phi_{m2} - \Phi_{m1}) = I\Delta \Phi_m$$

§ 10.6 带电粒子在磁场中的运动

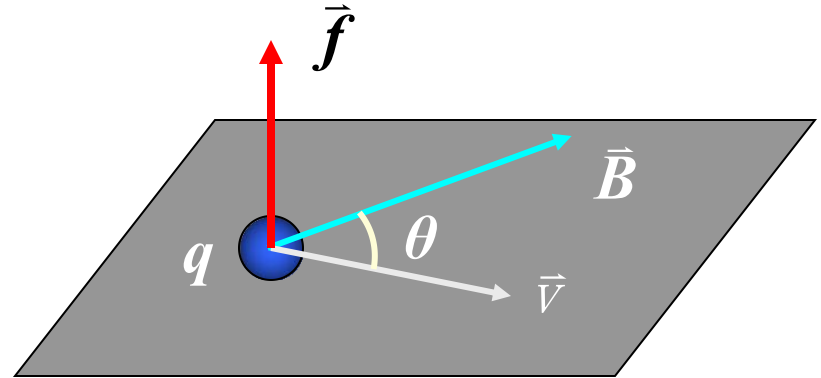
一. 洛伦兹力公式

- 实验结果

$$f \propto q, B, v, \sin \theta$$

$$|\vec{f}| = q v B \sin \theta$$

$$\boxed{\vec{f}_m = q \vec{v} \times \vec{B}}$$



★ 讨论

- (1) 洛伦兹力始终与电荷运动方向垂直，故 $\vec{f}$ 对电荷不做功
- (2) 在一般情况下，空间中电场和磁场同时存在

$$\boxed{\vec{F} = \vec{f}_e + \vec{f}_m = q\vec{E} + q\vec{v} \times \vec{B}}$$

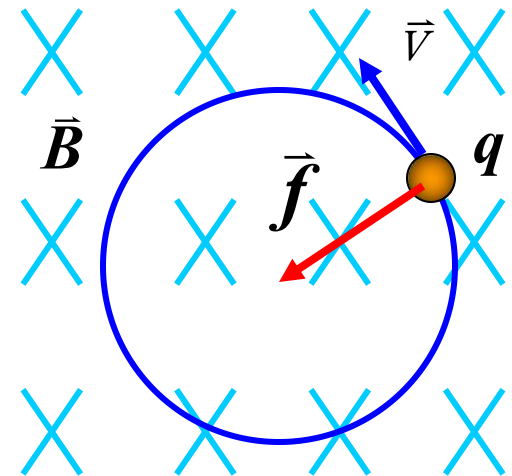
二. 带电粒子在均匀磁场中的运动

- $\vec{v} \perp \vec{B}$ 情况

$$q v B \sin \frac{\pi}{2} = m \frac{v^2}{R} \quad \longrightarrow \quad R = \frac{m v}{q B}$$

- 粒子回转周期与频率

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{q B} \quad \longrightarrow \quad f = \frac{q B}{2\pi m}$$



- 一般情况

$$v_{\parallel} = v \cos \theta$$

$$v_{\perp} = v \sin \theta$$

带电粒子作螺旋运动

$$R = \frac{m v_{\perp}}{qB} = \frac{m v \sin \theta}{qB}$$

$$h = v_{\parallel} T = \frac{2\pi m v \cos \theta}{qB}$$

